A Subspace Extraction Strategy for Many-objective Space Partitioning Optimization

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Abstract. This works extends a space partition search framework to divide the objective space into overlapping subspaces and propose a new conflict-based strategy to select the most conflicting subspaces for search. We test the effectiveness of the proposed strategy to improve the performance search of a multi-objective optimization algorithm on many-objective problems with non-redundant objectives.

1 Introduction

Space partitioning[1] has been proposed to improve the performance of conventional MOEAs[2] in many-objective problems. This approach dynamically splits the original high dimensional objective space into non-overlapping subspaces of lower dimensions and performs a concurrent search in the subspaces. Few strategies have been proposed to partition the space based on random subspace exploration[1] and on conflict information between objectives[3]. The space partitioning framework has also been extended to perform a combined search in the original space and in the subspaces[3].

In this work we extend further the framework so that the original space can be divided into overlapping subspaces and propose a new conflict-based strategy to select the subspaces for search.

2 Method

2.1 Space Partitioning Framework

The extended space partitioning framework repeats G iterations of a two-stage search process that consist of *integration* and *extraction* phases. In the *integration* phase an evolutionary multi-objective algorithm (MOEA) searches G_Φ generations in the original M-dimensional objective space $\Phi = \{f_1, f_2, \cdots, f_M\}$ of the problem. In the *extraction* phase N_S k-dimensional subspaces $X = \{\chi_1, \chi_2, \cdots, \chi_{N_S}\}$ are created from the objective space $\Phi = \{f_1, f_2, \cdots, f_M\}$ and the MOEA searches concurrently G_X generations in all of them. Population sizes for the integration and extraction phases are $|P|$ and $|P|/N_S$ (in each subspace), respectively. When the subspaces are created for the second stage, the MOEA selects the best $|P|/N_S$ solutions for each subspace from P. Similarly, when the second stage finishes, all solutions found in the subspaces are joined to form the new population P for the integration phase of the next iteration. The total number of generations is $G \times (G_\Phi + G_X)$. In this work we use NSGA-II[2] as the underlying MOEA for the integration and extraction phase.

2.2 Proposed Subspace Extraction Based on Conflict Information

The proposed method to extract a subspace based on conflict information creates all possible combinations of objectives that can form a subspace, order them by degree of conflict, and select those subspaces with the highest degree of correlation. Thus, in the proposed method one or more objectives could appear in more than one of the selected subspaces. The proposed method is as follows.

Step 1 Calculate the correlation $r_{f_i,f_j} \in [-1.0, 1.0]$ between each pair of objectives using population P and create the correlation matrix R.
Step 2 Create from R a conflict matrix C, where $c_{f_i,f_j} = -(r_{f_i,f_j} - 1.0) \in [0.0, 2.0]$. Thus, the degree of conflict increases from 1.0 to 2.0 with negative correlation and decreases from 1.0 to 0.0 with positive correlation.
Step 3 Create a S_{table} with all combinations ${}_M C_k$ of M objectives taken k at the time, $k \in [2, M-1]$. Each combination represents a possible subspace.
Step 4 Calculate the degree of conflict for each subspace in S_{table} by adding the degree of conflict c_{f_i,f_j} of all ${}_k C_2$ combinations of k objectives in the subspace taken 2 at the time.
Step 5 Sort the subspaces in S_{table} by descending order of degree of conflict and select the top N_S subspaces, $X = \{\chi_1, \chi_2, \cdots, \chi_{N_S}\}$.

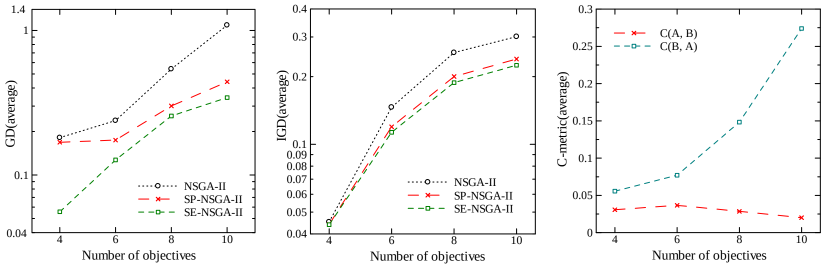
2.3 Conventional Space Partitioning Based on Conflict Information

In the conventional partition method based on conflict information[3], all objective functions are assigned to one and only one of the subspaces. In addition, partitions are created by repetitively finding the subspace with lowest conflict between objectives not yet assigned to a partition. Thus the first partition contains the subspace with the lowest conflict and the last partition the subspace with the highest conflict. The conventional strategy is as follows.

Step 1 $\sim$ **2** Same as proposed method.
Step 3 Initialize the set of objectives to be partitioned as $W = \{f_1, f_2, \ldots, f_M\}$.
Step 4 Create a neighborhood for each objective in W. The neighborhood of objective f_i is determined by itself and its $k-1$ nearest-neighbors objectives using the values $c_{i,j}$ of the conflict matrix C as a distance.
Step 5 Select the most compact neighborhood as a subspace.
Step 6 Eliminate the objectives in the selected subspace from W. Repeat from **Step 4** until all subspaces have been formed and W becomes empty.

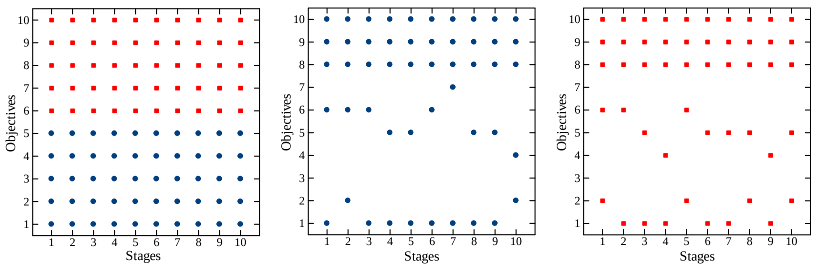
3 Simulation Results and Discussion

In this work we use DTLZ2_{BZ} as test problem. In this problem all objective functions are correlated. It is known that in the true Pareto Optimal set (POS) the second half of objective functions are negatively and strongly correlated to other functions. In this work we set the number of objectives to $M = \{4, 6, 8, 10\}$,



(a) *GD* (b) *IGD* (c) C-metric

Fig. 1. *GD*, *IGD*, and C-metric after 200 generations for each number of objectives



(a) Subspaces ψ_1, ψ_2 (b) Subspace χ_1 (c) Subspace χ_2

Fig. 2. Subspaces generated using $k = 5, N_S = 2$ with $M = 10$

the number of distance related variable to $l = 20$, and the total number of design variables to $n = l + M - 1 = M + 19$. To evaluate the obtained POS we use Generational Distance (GD), Inverted GD (IGD), and the C-metric. Results show the average of these measures on 30 runs of the algorithms with different random seeds. As genetic operators we use SBX crossover and Polynomial Mutation [2]. Population size is $|P| = 2000$, number of subspaces $N_S = 2$, and size of the subspace $k = M/N_S$. Thus, each subspace population size is $|P_S| = 1000$. The number of generations is $G_\Phi = 6(30\%)$ for the integration phase and $G_X = 14(70\%)$ for the extraction phase. The number of iterations is $G = 10$. Thus, the overall number of generations is $(G_\Phi + G_X) \times G = 200$.

We compare the performance of NSGA-II[2], NSGA-II with conventional space partitioning based on conflict information (SP-NSGA-II)[3], and NSGA-II with the proposed subspace extraction based on conflict information (SE-NSGA-II). **Fig.1** shows GD, IGD, and C-metric by the three algorithms at the final generation. First, from **Fig.1**(a) note that GD increases with the number of objectives, revealing that convergence becomes difficult for all algorithms. Note however that for any number of objectives both SP-NSGA-II and SE-NSGA-II lead to smaller GD than NSGA-II and that the proposed SE-NSGA-II achieves significantly smaller GD than SP-NSGA-II. Second, from **Fig.1**(b) note that a similar trend can be observed for IGD, although the difference between SE-NSGA-II and proposed SP-NSGA-II is smaller. This shows that the proposed method can achieve superior solutions in terms of convergence and diversity than the conventional space-partitioning algorithm based on conflict information. Next, **Fig.1**(c) shows the C-metric values C(A,B) and C(B,A), where A is the conventional SP-NSGA-II and B the proposed SE-NSGA-II algorithm. From C(B,A) note that the fraction of solutions of the conventional algorithm dominated by the proposed algorithm increases with the number of objectives, becoming almost 30% for 10 objectives. This shows that from the Pareto dominance relation standpoint the POS obtained by proposed method are superior.

Fig.2(a) shows the objectives included in each subspace by the conventional SP-NSGA-II for $M = 10$. The subspace ψ_1 in blue circle is the low conflict subspace and ψ_2 in red square is the high conflict subspace. From this figure note that every time the space is partitioned the same subspaces are selected. Similarly, **Fig.2** (b) and (c) show the subspaces χ_1 and χ_2 with the highest and second highest conflict, respectively, extracted by the proposed SE-NSGA-II. Note that both subspaces χ_1 and χ_2 are different and contain objectives f_8, f_9, f_{10} that are highly and negatively correlated to other functions. The proposed method allows the same objective function to be part of various highly conflicting subspaces, which helps the proposed method to search effectively in the subspaces that can contribute the most to the trade-off of the POS. In the conventional method, one of the spaces has the lowest correlation of all possible subspaces and thus the contribution by searching on it is limited.

4 Conclusions

This work has proposed a new strategy to extract subspaces based on conflict information for space partitioning algorithms. The proposed strategy is based on conflict information between objectives and allows overlapping between selected partitions. This allows the proposed strategy select the subspaces with highest degree of conflict between objectives. The proposed strategy was compared with the conventional strategy based on conflict information using NSGA-II in the space partitioning framework. Experimental results on $\mathrm{DTLZ2_{BZ}}$ problem showed the superiority of the proposed strategy finding Pareto optimal solutions with better convergence and diversity properties on problems with up to 10 objectives. In the future we would like to analyze the proposed strategy with various subspaces sizes and number of subspaces. Also, we would like to use the proposed strategy in other algorithms and problems.

References

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